

# EE 435

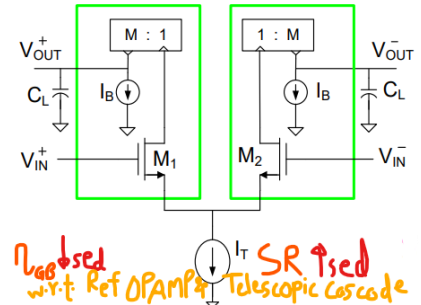
## Lecture 13

Cascaded Amplifiers

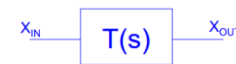
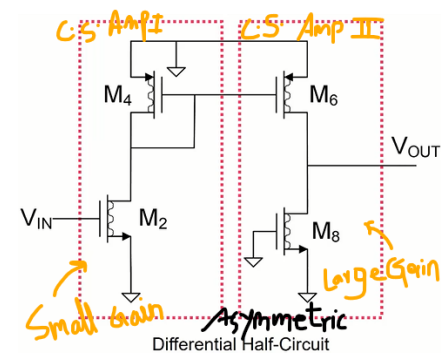
Two-Stage Op Amp Design

# Executive Summary Lecture 11

Thanks Akash



- Gain Enhancement structures; their GBW, SR.
- Stability, BIBO, location of poles of amplifier; requirement for stability = poles in Left Hand Plane and huge separation between poles on real axis.
- Dead network, its usefulness in obtaining poles, stability of opamp
- 2 Common source amplifier; gain determination.
- Absolute stability; Cascading multiple amplifier increases gain but decreases stability; Need to analyze complete system using different feedback structures. Use of unstable sub-system in a larger system can lead to useful applications.



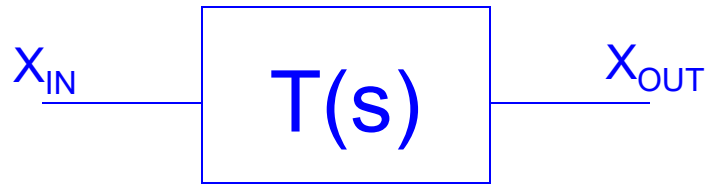
If  $T(s) = \frac{N(s)}{D(s)}$  is the transfer function of a linear system

Roots of  $N(s)$  are termed the zeros

Roots of  $D(s)$  are termed the poles

## Review of Basic Concepts

## Review from Last Time



If  $T(s) = \frac{N(s)}{D(s)}$  is the transfer function of a linear system

Roots of  $N(s)$  are termed the zeros

Roots of  $D(s)$  are termed the poles

**Theorem:** A linear system is stable iff all poles lie in the open left half-plane

- If a circuit is unstable, the output will either diverge to infinity or oscillate even if the input is set to 0
- A FB amplifier circuit that is not stable is not a useful “stand alone” FB amplifier
- A FB amplifier circuit that is “close” to becoming unstable is not a useful “stand alone” amplifier
- An amplifier circuit that exhibits excessive ringing or gain peaking is not a useful “stand alone” amplifier

## Routh-Hurwitz Stability Criteria:

A third-order polynomial  $s^3+a_2s^2+a_1s+a_0$  has all poles in the LHP iff all coefficients are positive and  $a_1a_2>a_0$

- Very useful in amplifier and filter design
- Can easily determine if poles in LHP without finding poles
- But tells little about how far in LHP poles may be
- RH exists for higher-order polynomials as well

# Cascaded Amplifier Issues

For identical first-order lowpass stage gains  $A = \frac{A_0 \tilde{p}}{s + \tilde{p}}$

Summary:

- Three amplifier cascades - for ideally identical stages  $8 > \beta A_0^3$ 
  - seldom used in industry though some recent products use this method !
  - invariably modify A
- Four or more amplifier cascades - problems even larger than for three stages
  - seldom used in industry !

Consider now two amplifiers in cascade

## Review from Last Time

Two-stage Cascade (continued)

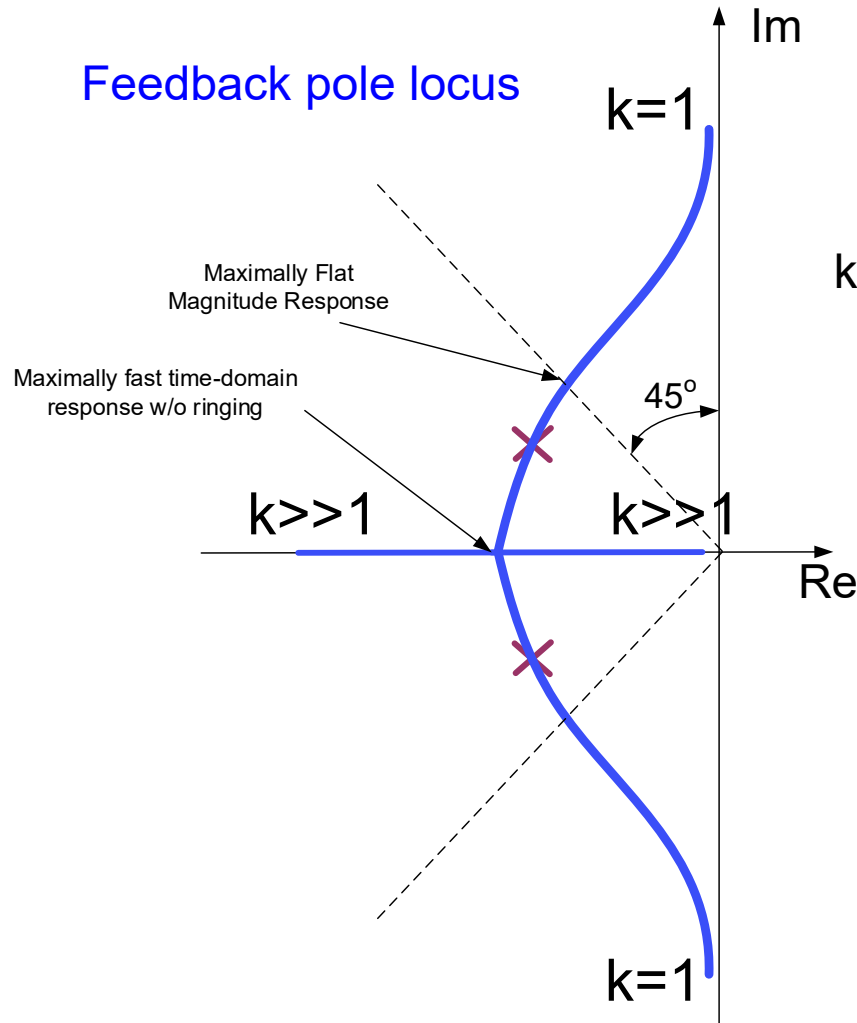
$$D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{OTOT})$$

$$A_1 = \frac{A_{01} \tilde{p}_1}{s + \tilde{p}_1}$$

$$A_2 = \frac{A_{02} k \tilde{p}_1}{s + k\tilde{p}_1}$$

$$A_{OL} = \prod_{i=1}^2 A_k$$

$k$  is the open-loop pole spread



Will be shown that maximally flat response for second-order all-pole amplifier occurs with  $\theta=45^\circ$  and maximally fast step response w/o ringing occurs with  $\theta=90^\circ$

## Review of Basic Concepts

### Review from Last Time

Consider a second-order factor of a denominator polynomial,  $P(s)$ , expressed in integer-monic form

$$P(s) = s^2 + a_1s + a_0$$

Then  $P(s)$  can be expressed in several alternative but equivalent ways

$$s^2 + s \frac{\omega_0}{Q} + \omega_0^2$$

$$s^2 + s2\zeta\omega_0 + \omega_0^2$$

$$(s - p_1)(s - p_2)$$

if real – axis poles

$$(s - p_1)(s - kp_1)$$

and if complex conjugate poles,

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$

$$(s + re^{j\theta})(s + re^{-j\theta})$$

Widely used alternate parameter sets:

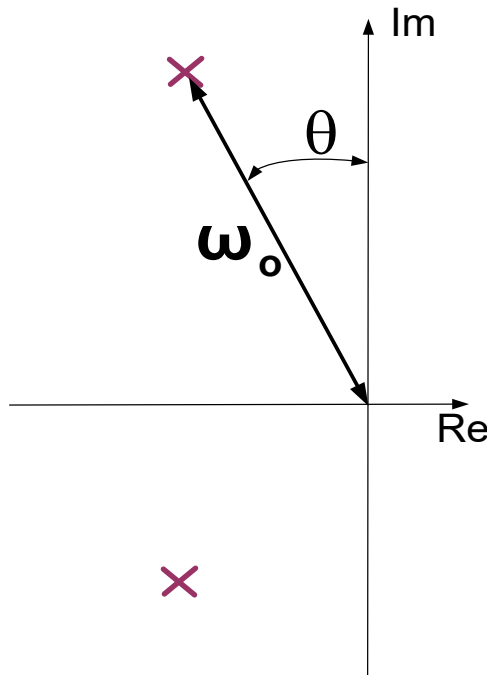
$$\{ (a_1, a_2) (\omega_0, Q) (\omega_0, \zeta) (p_1, p_2) (p_1, k) (\alpha, \beta) (r, \theta) \}$$

These are all 2-parameter characterizations of the second-order factor and it is easy to map from any one characterization to any other

## Review from Last Time

### Review of Basic Concepts

$$s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{0TOT}) \quad \Rightarrow \quad s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$



$$\sin\theta = \frac{1}{2Q}$$

$\omega_0$  = magnitude of pole

$Q$  determines the angle of the pole

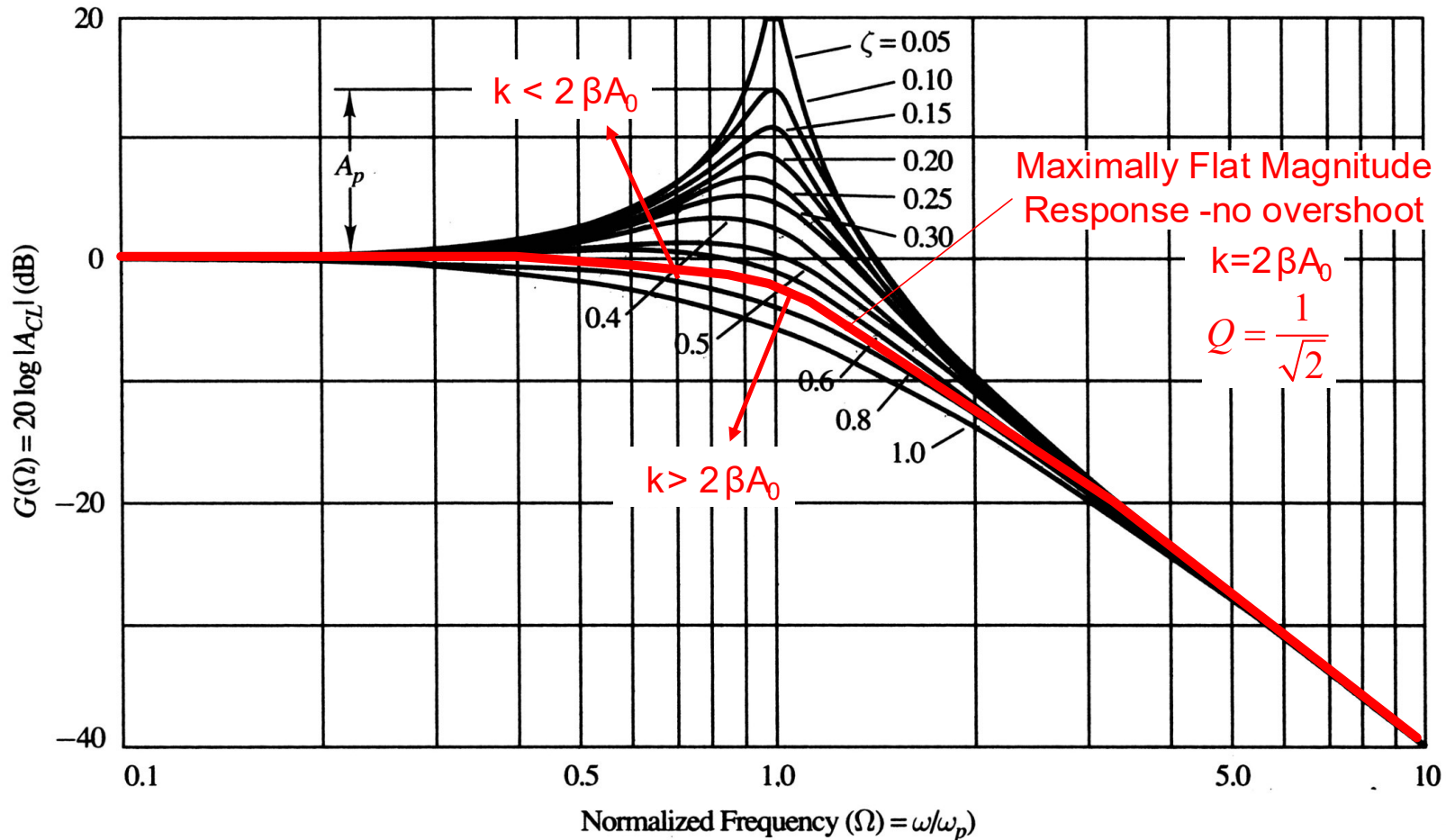
Observe:  $Q=0.5$  corresponds to two identical real-axis poles  
 $Q=.707$  corresponds to poles making  $45^\circ$  angle with Im axis



## Review from Last Time

### Magnitude Response of 2<sup>nd</sup>-order all-pole (Low-pass) Function

$$Q = \frac{1}{2\xi}$$



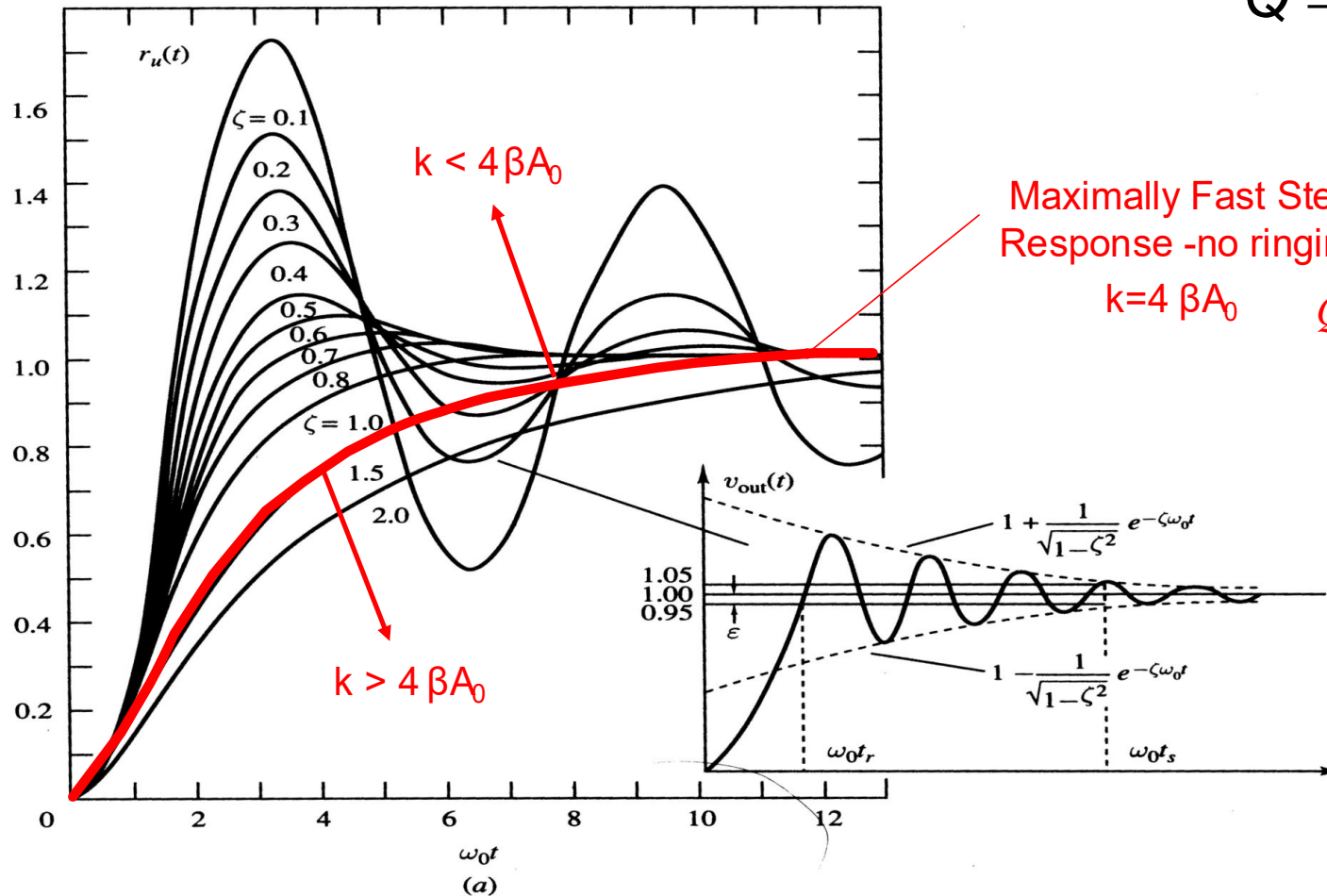
From Laker-Sansen Text

For two-stage all-pole amplifiers, must have open-loop pole spread,  $k$ , very large to avoid overshoot in closed-loop gain

# Review from Last Time

## Step Response of 2<sup>nd</sup>-order all-pole (Low-pass) Function

$$Q = \frac{1}{2\xi}$$



$Q_{MAX}$  for no overshoot = 1/2

From Laker-Sansen Text

For two-stage amplifiers, must have open-loop pole spread,  $k$ , very large to avoid ringing in step response

## Review from Last Time

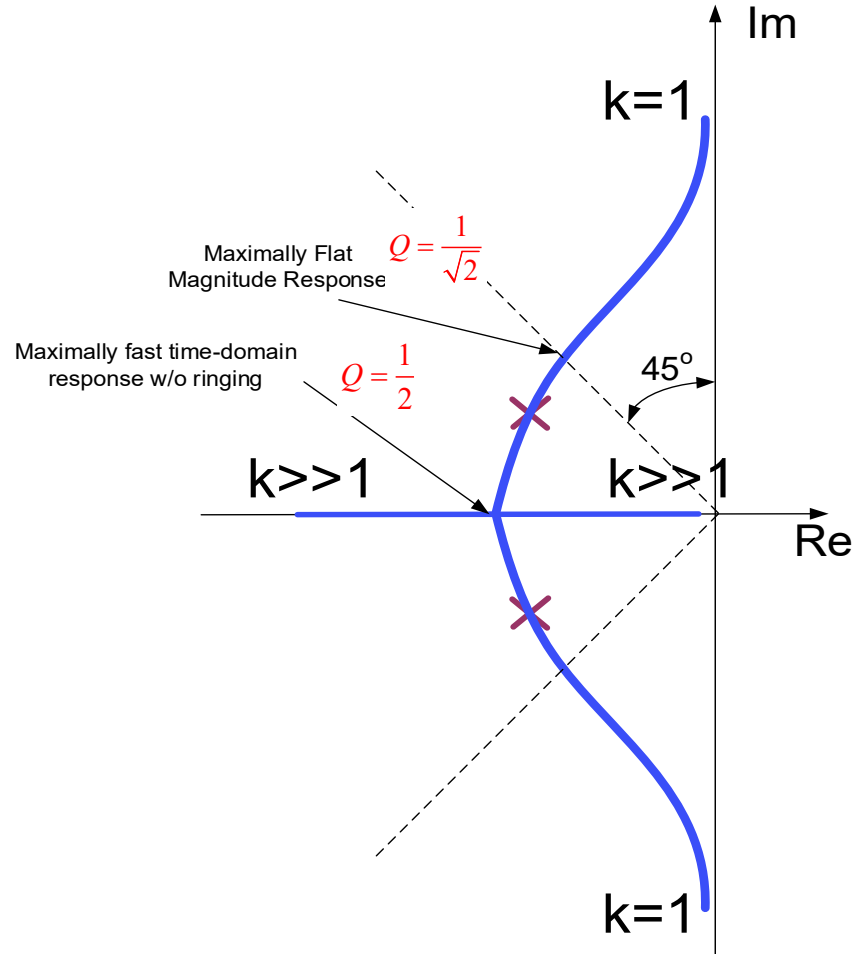
Two-stage Cascade second-order (continued)

$$D_{FB}(s) = s^2 + s\tilde{\omega}_1(1+k) + k\tilde{\omega}_1^2(1+\beta A_{OTOT})$$

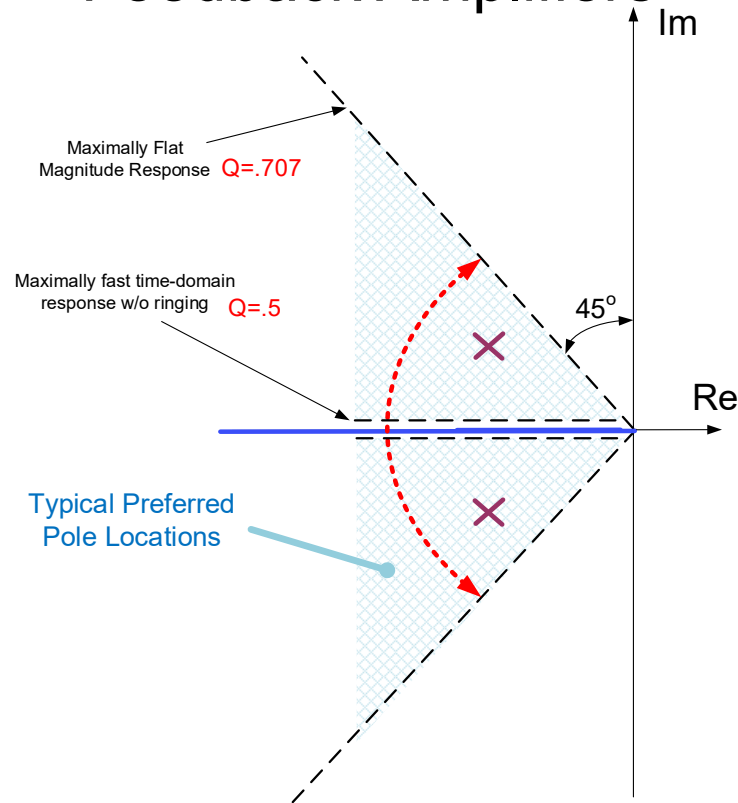
Alternate notation for  $D_{FB}(s)$

$$D_{FB}(s) = s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$

Feedback pole locus

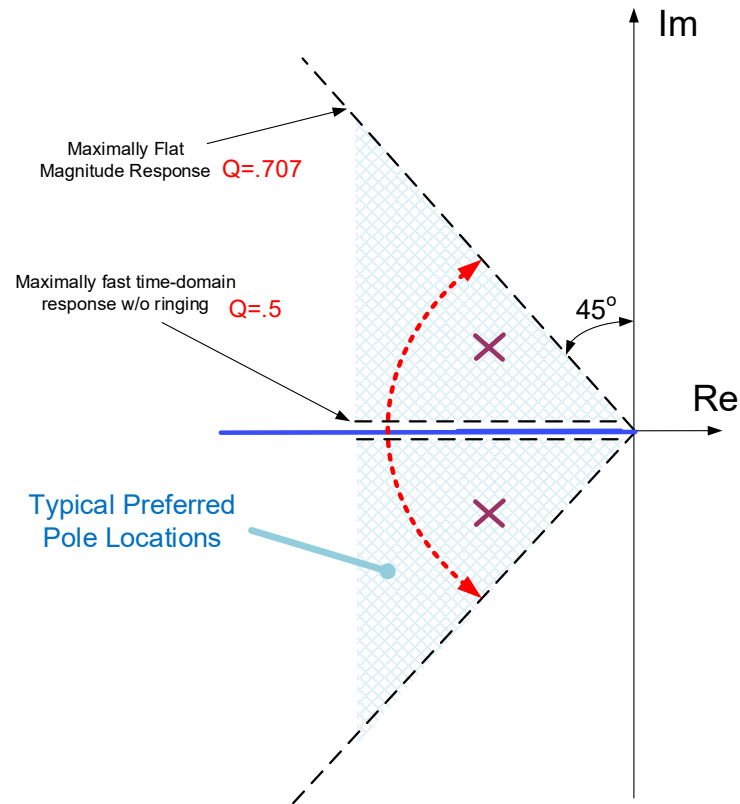


# Typical Target Closed-loop Pole Locations for Feedback Amplifiers



- For two-stage all-pole amplifiers, must have open-loop pole spread,  $k$ , very large to obtain desired performance of feedback amplifier
- Cascading of two identical amplifier stages to increase op amp gain not practical
- Two-stage amplifiers widely used to build op amps but must manage pole spreads (even if not all-pole) - this will be discussed in detail when on the topic of compensation

# Determination of open-loop pole spread, $k$ , for acceptable feedback performance



# Determination of desired open-loop pole spread, $k$ ,

Two-stage Cascade second-order all pole

$$A_1 = \frac{A_{01} \tilde{p}_1}{s + \tilde{p}_1}$$

$$D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{0TOT}) \quad \tilde{p}_2 = k\tilde{p}_1 \quad A_2 = \frac{A_{02} \tilde{p}_2}{s + \tilde{p}_2} = \frac{A_{02} k\tilde{p}_1}{s + k\tilde{p}_1}$$

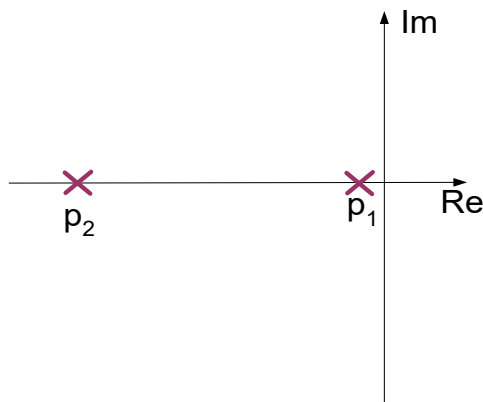
From quadratic equation (for  $k$  large and  $\beta A_{0TOT} \gg 1$ ) obtain:

$$p_{1F,2F} \cong \frac{\tilde{p}_1}{2} \left( -k \pm j\sqrt{4A_{0TOT}k\beta - k^2} \right) \quad Q = \frac{\sqrt{k}}{(1+k)} \sqrt{\beta A_{0TOT}} \underset{k \text{ large}}{\cong} \sqrt{\frac{\beta A_{0TOT}}{k}}$$

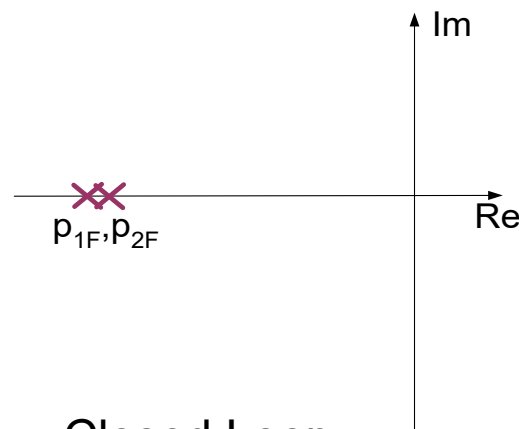
Or equivalently  $k \underset{k \text{ large}}{\cong} \frac{\beta A_{0TOT}}{Q^2}$

Case 1: Identical negative real-axis poles (no zeros); must make discriminate 0, thus (maximally fast time-domain step response w/o ringing)

$$k \cong 4\beta A_{0TOT} \implies Q = \frac{1}{2}$$



Open Loop



Closed Loop

# Determination of open-loop pole spread, k,

Two-stage Cascade second-order all pole (continued)

$$p_{1,2} \cong \frac{\tilde{p}_1}{2} \left( -k \pm j\sqrt{4A_{0TOT}k\beta - k^2} \right)$$

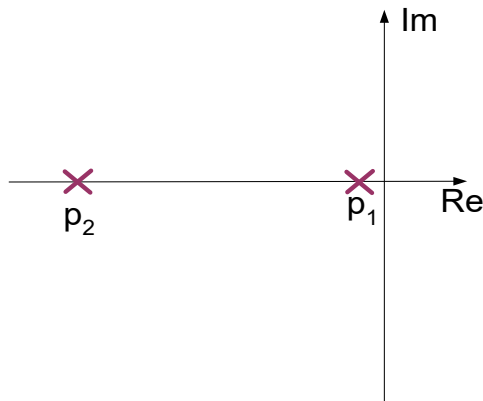
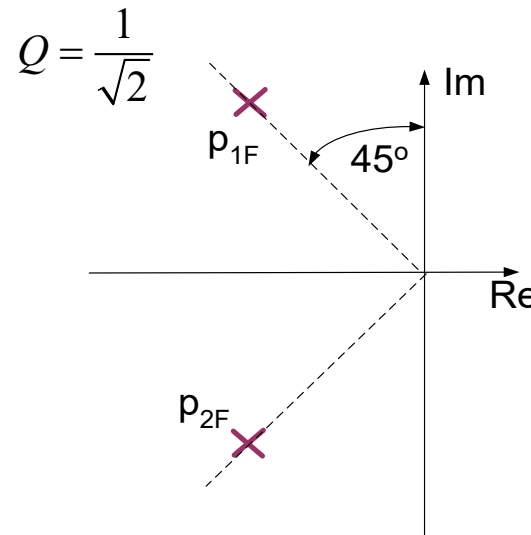
$$\tilde{p}_2 = k\tilde{p}_1$$

$$Q \underset{k \text{ large}}{\cong} \sqrt{\frac{\beta A_{0TOT}}{k}}$$

Case 2: Maximally flat all-pole magnitude response; must make real and imaginary parts equal

$$k = \sqrt{4A_{0TOT}k\beta - k^2}$$

$$k \cong 2\beta A_{0TOT}$$



Open Loop

Closed Loop

- Small ringing in step response
- Factor of 2 reduction in pole spread

# Determination of open-loop pole spread, k,

Two-stage Cascade second-order all pole(continued)

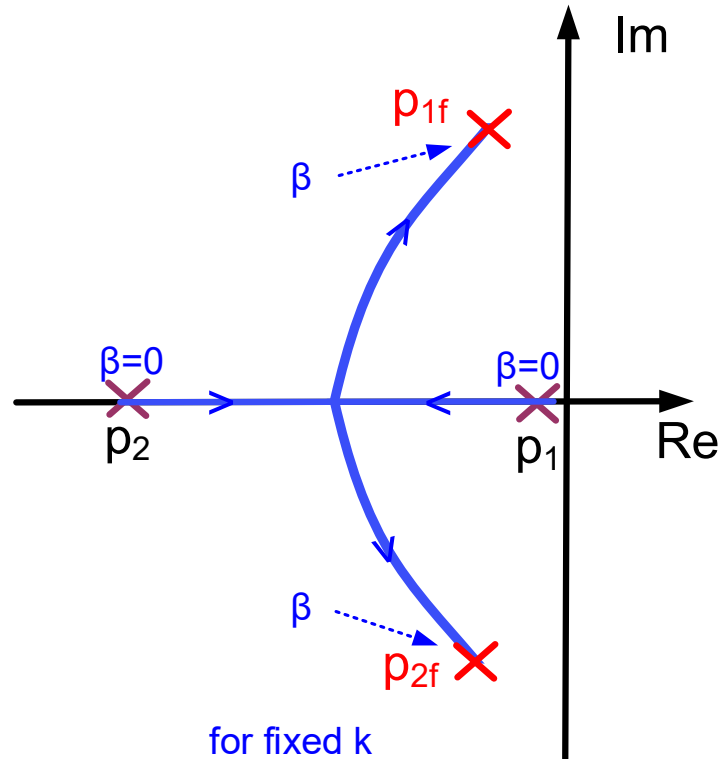
$$p_{1,2} \cong \frac{\tilde{p}_1}{2} \left( -k \pm j\sqrt{4A_{0TOT} k\beta - k^2} \right) \quad \tilde{p}_2 = k\tilde{p}_1$$

- The pole spread for maximal frequency domain flatness or fast non-ringing time domain response is quite large for the two-stage amplifier but can be achieved
- Usually will make angle of feedback poles with imaginary axis between 45° and 90°
- This results in an open loop pole spread that satisfies the relationship
$$4\beta A_{0TOT} > k > 2\beta A_{0TOT}$$
- “Compensation” is the modification of the pole locations of an amplifier to achieve a desired closed-loop pole angle or pole placement
- “Compensation” should not be considered as a modification of the pole locations to achieve stability since an amplifier is of little use if stability concerns are present



## Brief Summary of Operation of cascade of 2 single-pole amplifiers

# Pole Locus vs $\beta$ for 2<sup>nd</sup>-order Amplifiers



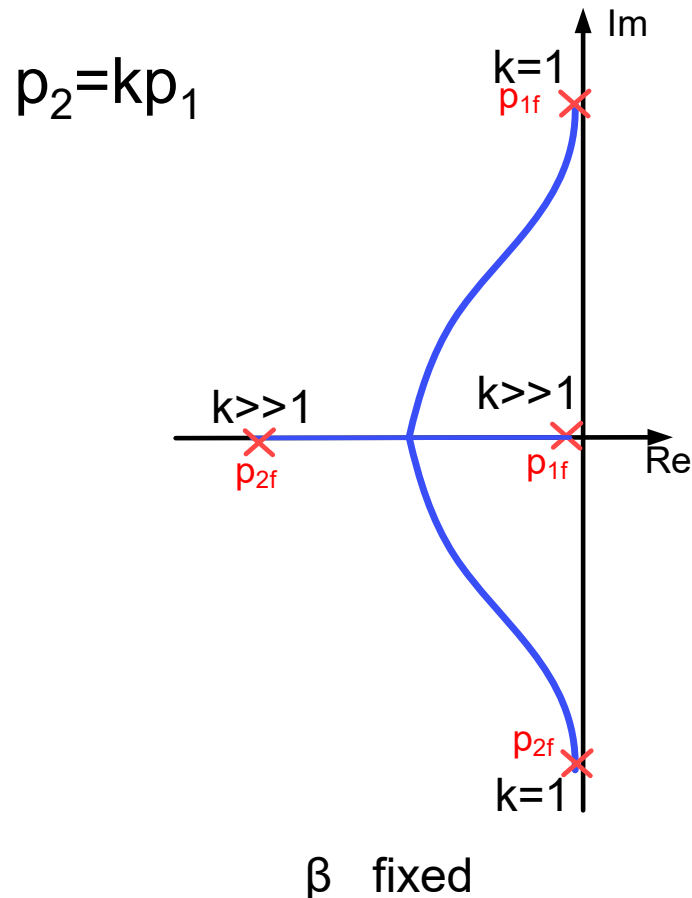
$\beta=0$  : open loop poles  $p_1, p_2$

Actual  $\beta$  : closed loop poles  $p_{1f}, p_{2f}$

- Feedback poles strongly dependent upon  $\beta$
- $\beta$  large (i.e.  $\beta=1$ ) often most challenging to deal with

## Brief Summary of Operation of cascade of 2 single-pole amplifiers

# Closed-loop pole Locus vs $k$ for 2<sup>nd</sup>-order Amplifiers

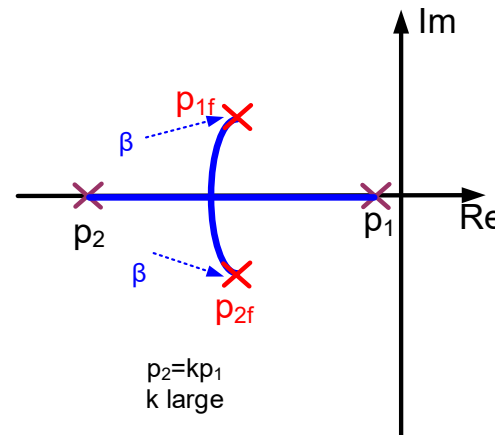
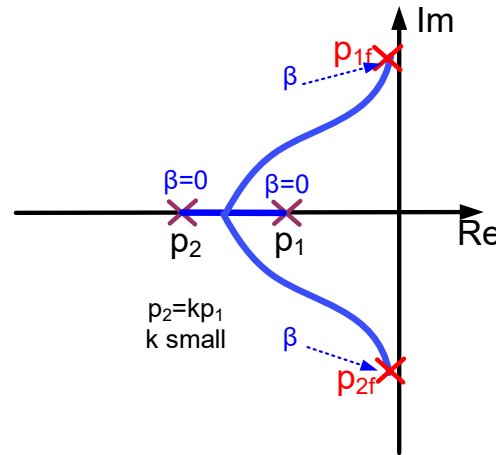


- poles strongly dependent upon open-loop pole spread,  $k$
- $k$  small often most challenging to deal with

## Brief Summary of Operation of cascade of 2 single-pole amplifiers

# Pole Locus vs $\beta$ for 2<sup>nd</sup>-order amplifiers for varying $k$

$$p_2 = kp_1$$



- poles strongly dependent upon open-loop pole spread,  $k$ , and  $\beta$
- $k$  small and  $\beta$  large are often most challenging to deal with

# Cascaded Amplifier Summary

$$A_1 = \frac{A_{01} \tilde{p}_1}{s + \tilde{p}_1}$$

$$A_2 = \frac{A_{02} \tilde{p}_2}{s + \tilde{p}_2} = \frac{A_{02} k \tilde{p}_1}{s + k \tilde{p}_1}$$

$$\tilde{p}_2 = k \tilde{p}_1$$

- Single-stage amplifiers
  - widely used in industry, little or no concern about compensation
  
- Two amplifier cascades – for separated poles  $4\beta A_{0TOT} > k > 2\beta A_{0TOT}$ 
  - (both single pole)
  - widely used in industry but compensation is essential
  - spread dependent upon  $\beta$  and most stringent for large  $\beta$
  
- Three amplifier cascades - for ideally identical stages  $8 > \beta A_0^3$ 
  - (all single pole)
  - seldom used in industry !
  
- Three amplifier cascades - for separated poles
  - (all single pole)
  - $(1 + k_2 + k_3)(k_2 + k_3 + k_2 k_3) > \beta A_{0TOT}$
  - seldom used in industry but starting to appear but compensation essential!
  
- Four or more amplifier cascades - problems even larger than for three stages
  - seldom used in industry !

Note: Some amplifiers that are termed single-stage amplifiers in many books and papers are actually two-stage amplifiers and some require modest compensation. Some that are termed two-stage amplifiers are actually three-stage amplifiers. These invariably have a very small gain on the first stage and a very large bandwidth. The nomenclature on this summary refers to the number of stages that have reasonably large gain.

## Summary of Cascaded Amplifier Characteristics

A cascade of amplifiers can result in a very high dc gain !

Characteristics of feedback amplifier (where the op amp is applied) are of ultimate concern

Some critical and fundamental issues came up with even the most basic cascades when they are used in a feedback configuration

Must understand how open-loop and closed-loop amplifier performance relate before proceeding to design amplifiers by cascading

## Summary of Amplifier Characteristics

An amplifier is stable iff all poles lie in the open LHP

Routh-Hurwitz Criteria is often a practical way to determine if an amplifier is stable

Although stability of an amplifier is critical, a good amplifier must not only be stable but generally must satisfy magnitude peaking and/or settling requirements thus poles need to be moved a reasonable distance (in the angular sense) from the imaginary axis

The cascade of three identical high-gain all-pole amplifiers will result in a pole-pair far in the right half plane when feedback is applied so FB amplifier will be unstable

$$A = \frac{A_0 \tilde{p}}{s + \tilde{p}}$$


$$A_{\text{FB}} = \frac{A}{1 + A\beta} = \frac{A_0^3}{\left(\frac{s}{\tilde{p}} + 1\right)^3 + \beta A_0^3}$$

For stability

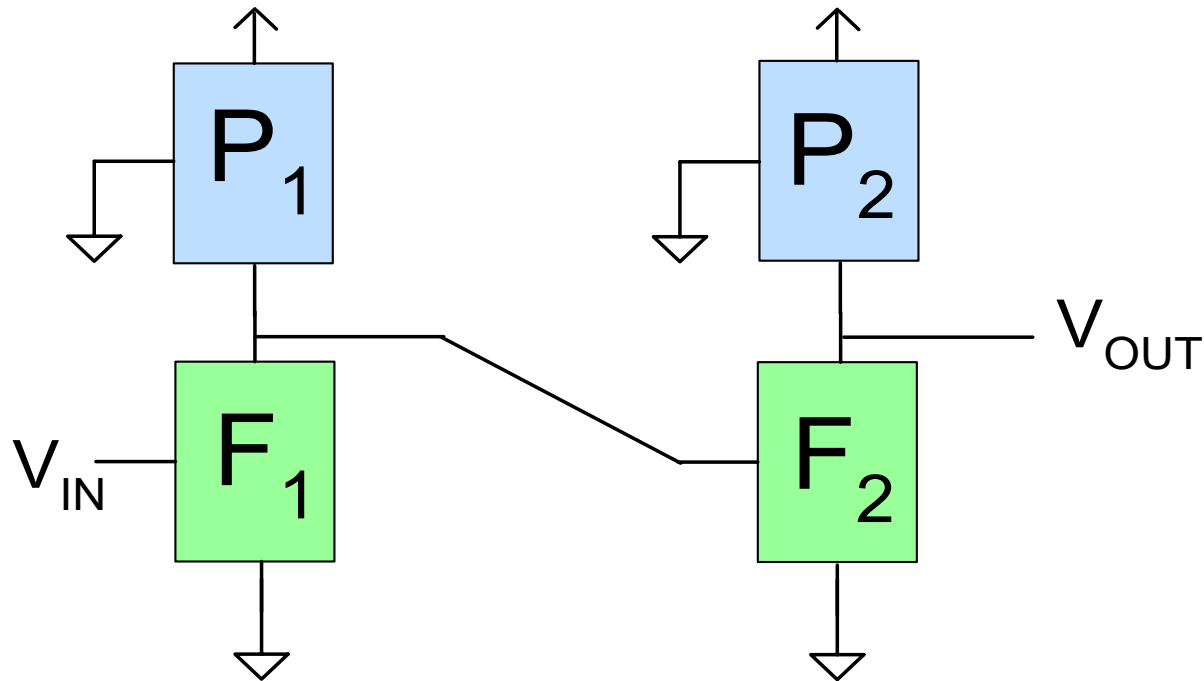
$$8 > \beta A_0^3$$

## Where we are at:

# Amplifier Design

- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Other Basic Gain Enhancement Approaches
  - Cascaded Amplifiers  
(will return to this later)
-  Two-Stage Op Amp
  - Compensation
  - Breaking the Loop
- Other Issues in Amplifier Design
- Summary Remarks

# Basic Two-Stage Cascade

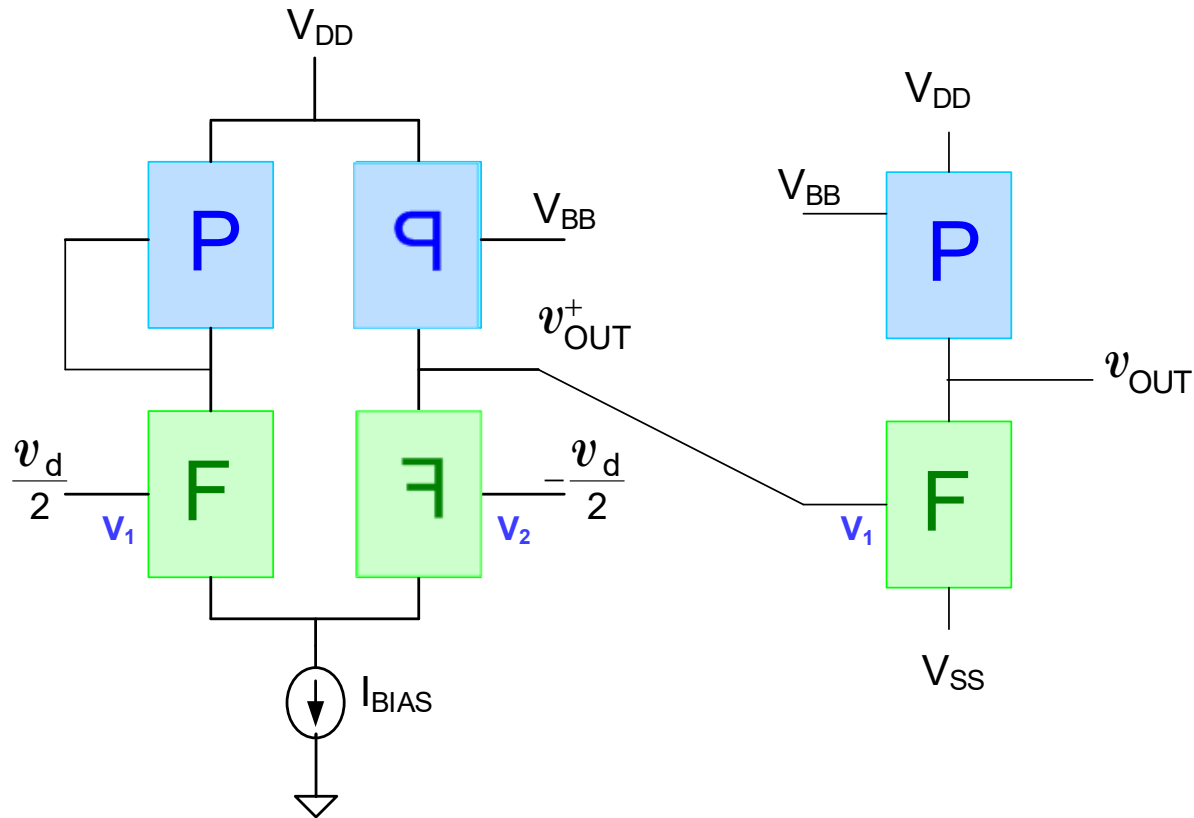


Can be extended to fully differential on first and/or second stage

- Simple Concept
- Several variants of basic cascade concept
- Must decide what to use for the two quarter circuits

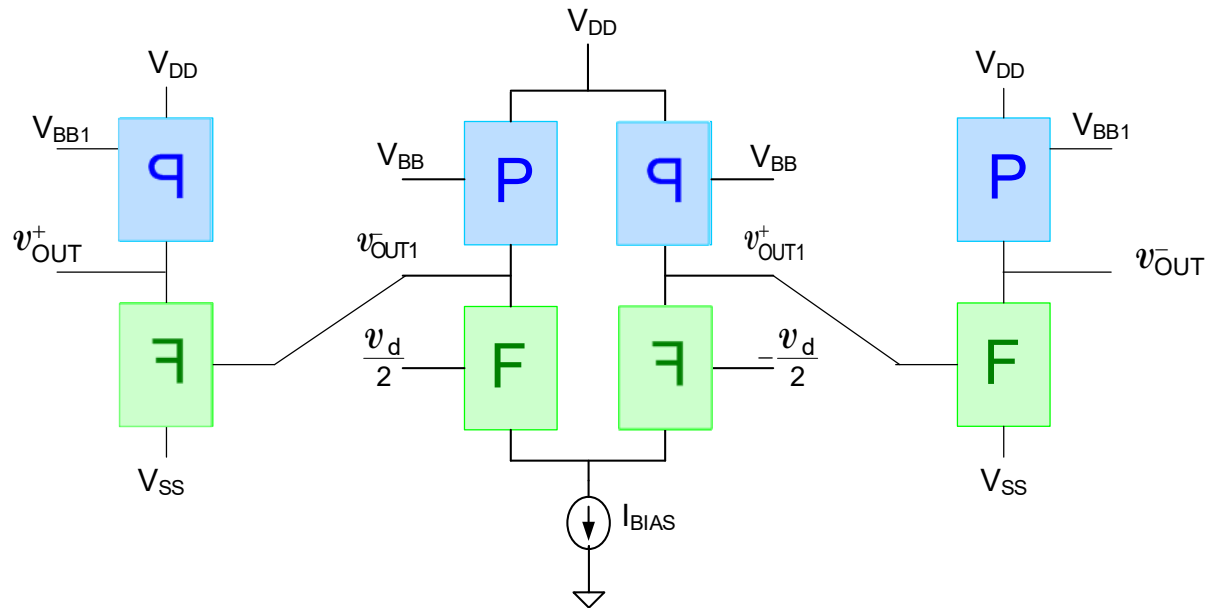


# Basic Two-Stage Cascade



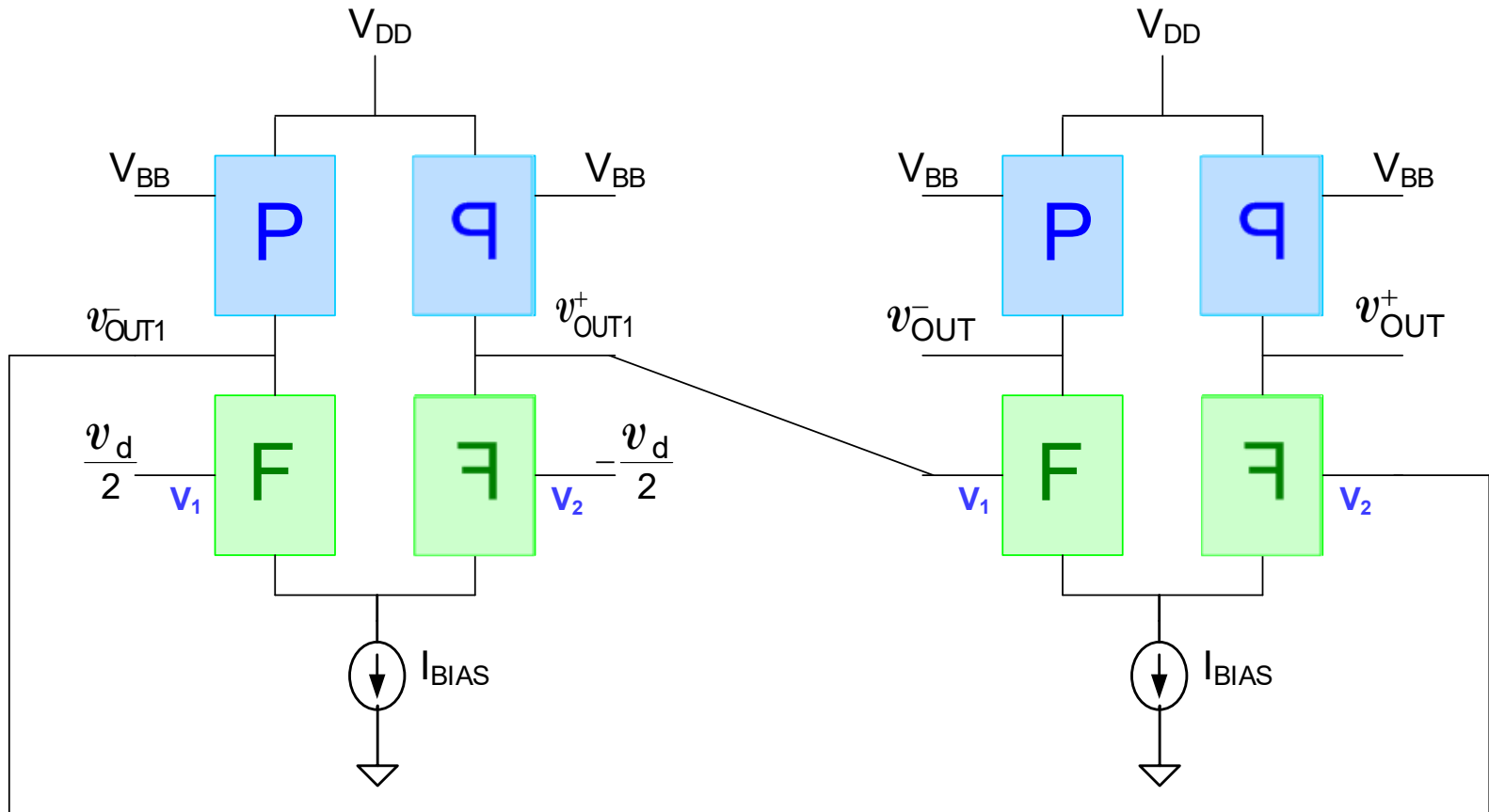
- Widely used structure for single-ended output
- Quarter circuits often different between first stage and second stage

# Basic Two-Stage Cascade



- Widely used structure for differential outputs
- Quarter circuits often different between first stage and second stage

# Basic Two-Stage Cascade



- Could be used but less popular

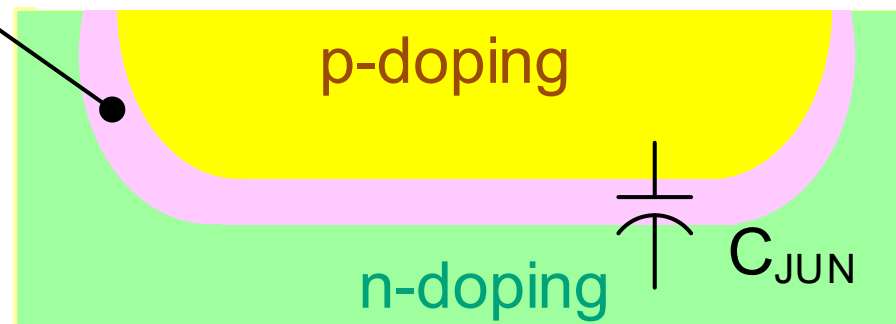
# Two-stage op amp design

It is essential to know where the poles of the op amp are located since there are some rather strict requirements about the relative location of the open-loop poles when the op amp is used in a feedback configuration.

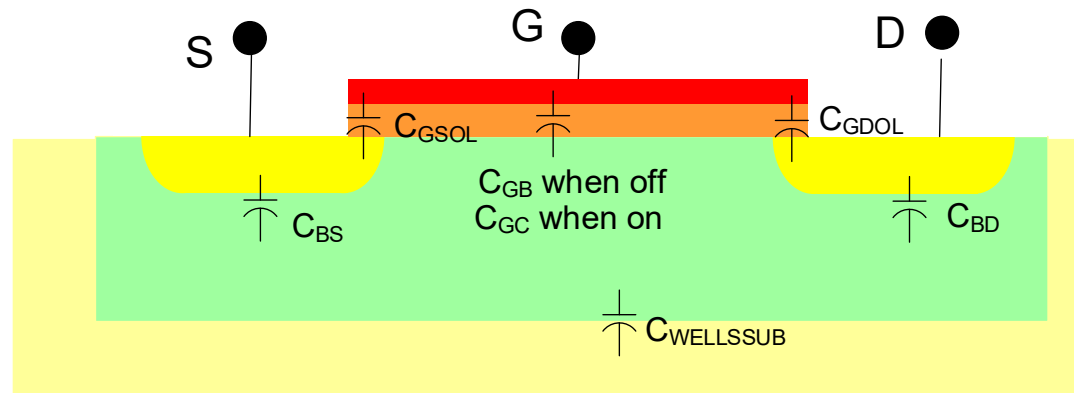
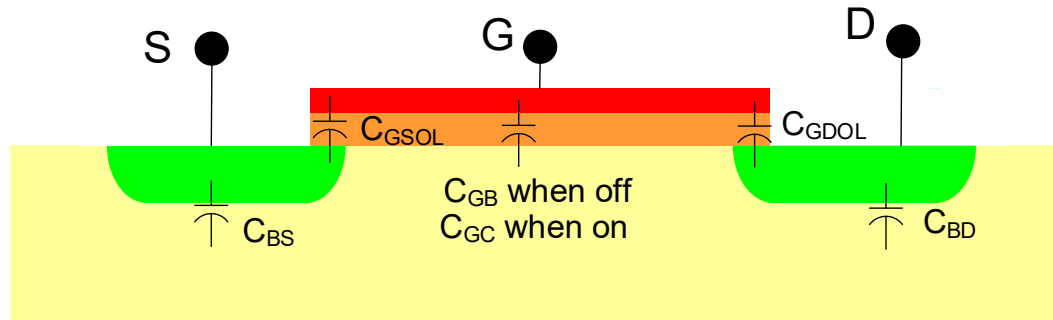
# Parasitic Capacitances in MOS Devices

- Depletion region is formed between reverse-biased pn junctions
- Creates a capacitance  $C_{JUN}$
- Voltage, area, and doping level dependent
- Can be quite large for large junctions

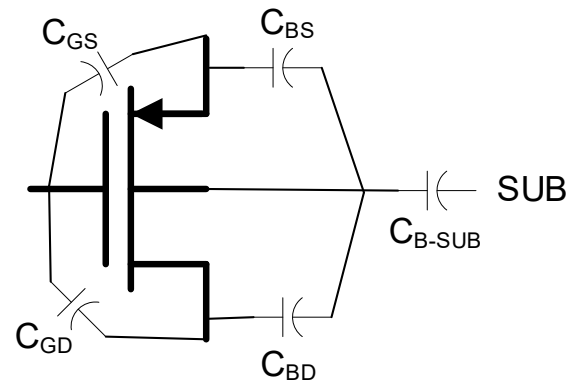
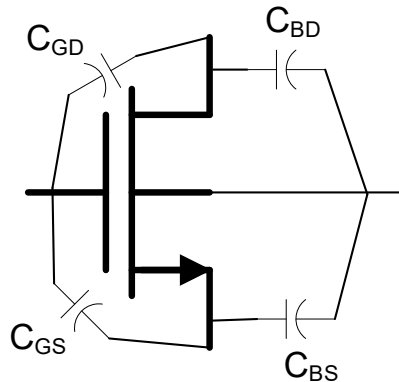
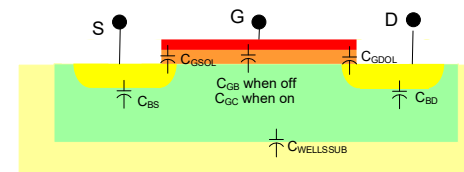
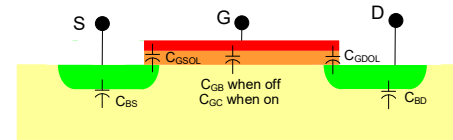
Depletion  
Region



# Parasitic Capacitances in MOS Devices

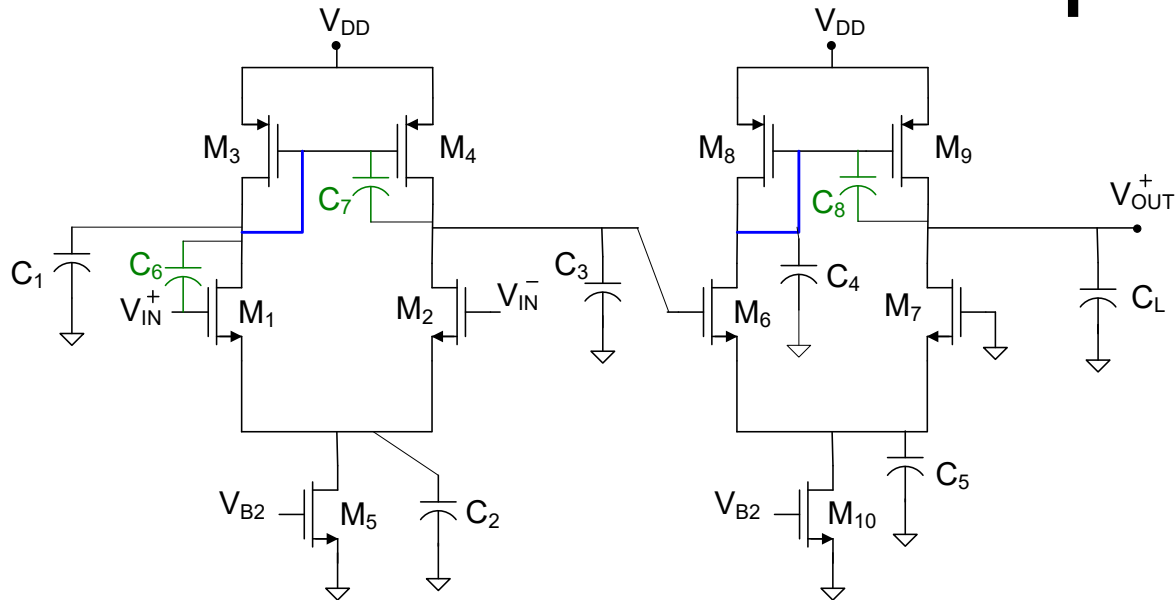


# Parasitic Capacitances in MOS Devices



- Parasitic Capacitances added to Device Models
- $C_{GS}$  is often largest
- $C_{BD}$  and  $C_{BS}$  often quite large with large drain/source area

# Poles and Zeros of Amplifiers



Cascaded Amplifier showing some of the capacitors

- There are a large number of parasitic capacitors in an amplifier (approx 5 for each transistor)
- Many will appear in parallel but the number of equivalent capacitors can still be large
- Order of transfer function is equal to the number of non-degenerate energy storage elements
- Obtaining the transfer function of a high-order network is a lot of work !
- Essentially every node in an amplifier has a capacitor to ground and these often dominate the frequency response of the amplifier (but not always)



# Pole approximation methods

1. Consider all shunt capacitors
2. Decompose these into two sets, those that create low frequency poles and those that create high frequency poles (large capacitors create low frequency poles and small capacitors create high frequency poles)  
 $\{C_{L1}, \dots, C_{Lk}\}$  and  $\{C_{H1}, \dots, C_{Hm}\}$
3. To find the  $k$  low frequency poles, replace all independent voltage sources with ss shorts and all independent current sources with ss opens, all high-frequency capacitors with ss open circuits and, one at a time, select  $C_{Lh}$  and determine the impedance facing it, say  $R_{Lh}$  if all other low-frequency capacitors are replaced with ss short circuits. Then an approximation for the pole corresponding to  $C_{Lh}$  is

$$p_{Lh} = -1/(R_{Lh} C_{Lh})$$

4. To find the  $m$  high-frequency poles, replace all independent voltage sources with ss shorts and all independent current sources with ss opens, replace all low-frequency capacitors with ss short circuits and, one at a time, select  $C_{Hh}$  and determine the impedance facing it, say  $R_{Hh}$  if all other high-frequency capacitors are replaced with ss open circuits. Then the approximation for the pole corresponding to  $C_{Hh}$  is

$$p_{Hh} = -1/(R_{Hh} C_{Hh})$$

# Pole approximation methods

These are just pole approximations but are often quite good

Provides closed-form analytical expressions for poles in terms of components of the network that can be managed during design

Provides considerable insight into what is affecting the frequency response of the amplifier

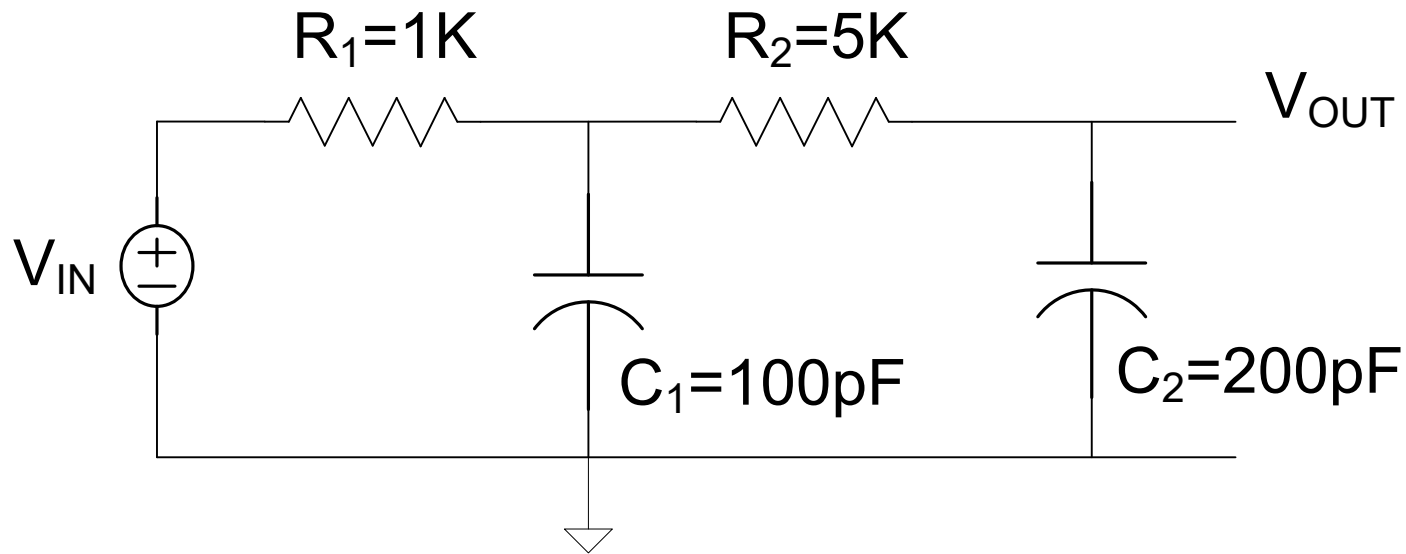
Pole approximation methods give no information about zero locations

Many authors refer to the “pole on a node” and this notation comes from the pole approximation method discussed on previous slide

Approach does a reasonable job of obtaining dominant low frequency poles (highest) and the dominant high frequency pole (lowest) if there is modest pole separation

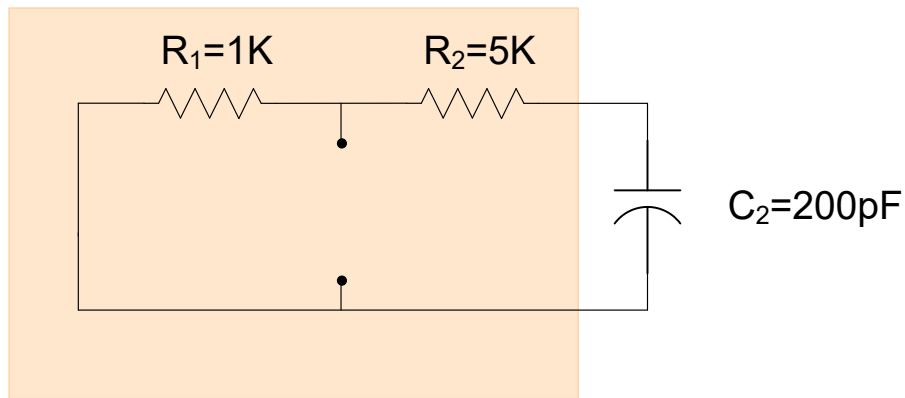
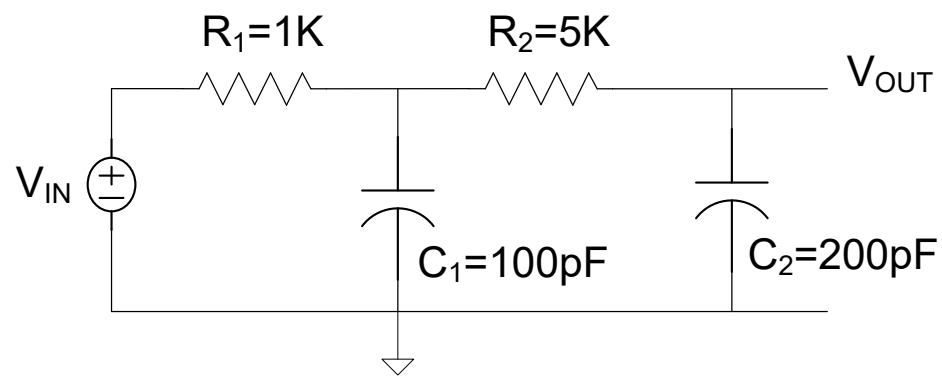
Dominant low frequency and dominant high frequency poles are often most important

Example: Obtain the approximations to the poles of the following circuit



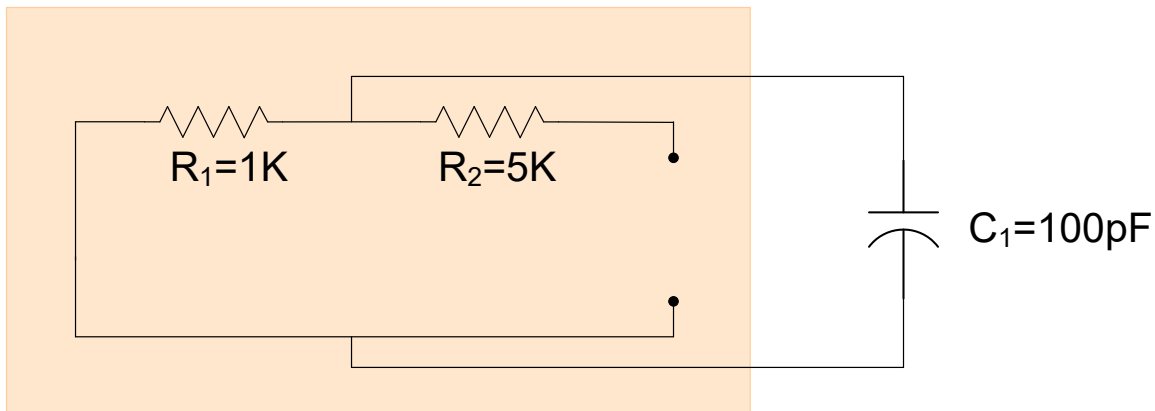
Since  $C_1$  and  $C_2$  are small, have two high-frequency poles

$\{C_1, C_2\}$



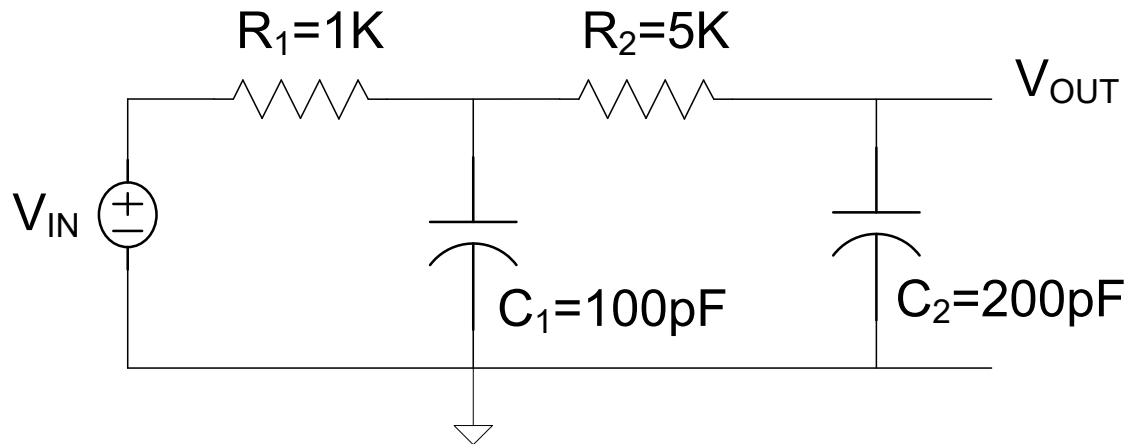
$$p_{H2} = - \frac{1}{C_2 (R_1 + R_2)}$$

$$p_{H2} = - 833Krad/sec$$



$$p_{H1} = - \frac{1}{C_1 R_1}$$

$$p_{H1} = -10M rad/sec$$



In this case, an exact solution is possible


$$T(s) = \frac{1}{R_1 R_2 C_1 C_2} \frac{1}{s^2 + \left[ \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} \right] s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$p_{H1} = -12.2M \text{ rad/sec} \quad (18\% \text{ error})$$

$$p_{H2} = -821K \text{ rad/sec} \quad (1.4\% \text{ error})$$

## Where we are at:

# Amplifier Design

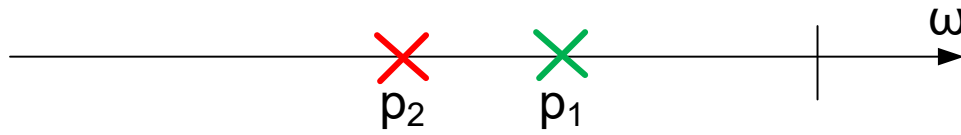
- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Other Basic Gain Enhancement Approaches
  - Cascaded Amplifiers  
(will return to this later)
- Two-Stage Op Amp
  -  – Compensation
  - Breaking the Loop
- Other Issues in Amplifier Design
- Summary Remarks

# Compensation of Two-Stage Cascade

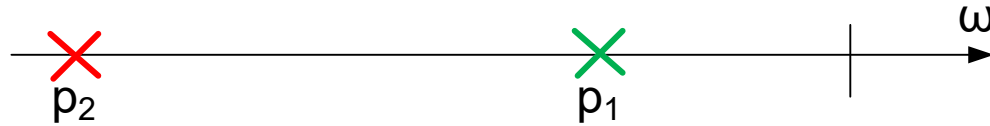
- “Compensation” is the modification of the op amp frequency response (that of the open-loop amplifier) so that acceptable ringing or overshoot or lack thereof in the closed-loop response is achieved
- Often do compensation for feedback amplifier applications though could compensate for closed-loop performance in other applications such as in a filter
- If two stages in cascade are first-order lowpass, compensation strategy is often to make an adequate pole spread to get acceptable closed-loop performance
- Often focus on the poles on the two nodes if cascade is of first-order lowpass stages
- If large spread of two poles that may inherently be close is required, can make one much larger or make one much smaller but fundamental speed limitations in a process often make it impossible to make one pole much larger so only alternative is often to make one pole much smaller

**Note:** Have intentionally not mentioned the term “stability” when discussing compensation

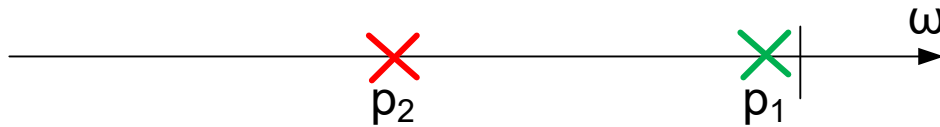
# Compensation Concepts



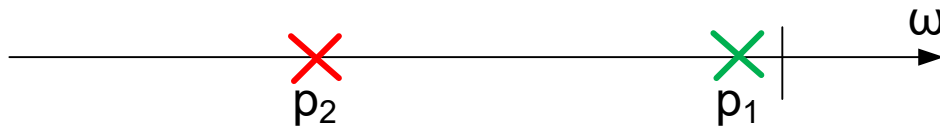
Original Pole Locations  
**Inadequate Separation**



Technology Speed  
Limitations



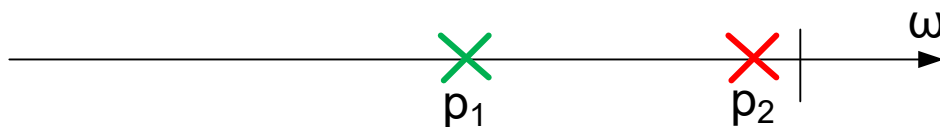
Most Widely Used Approach  
(but dramatically slows circuit)



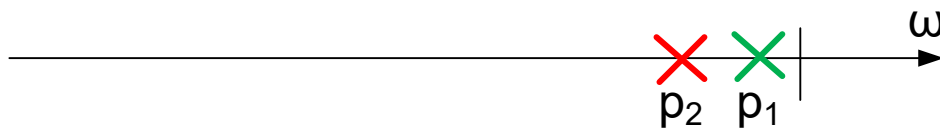
Modest left-movement of  $p_2$   
may be possible



Requires more pole  
movement



Modest left-movement of  $p_1$   
may be possible

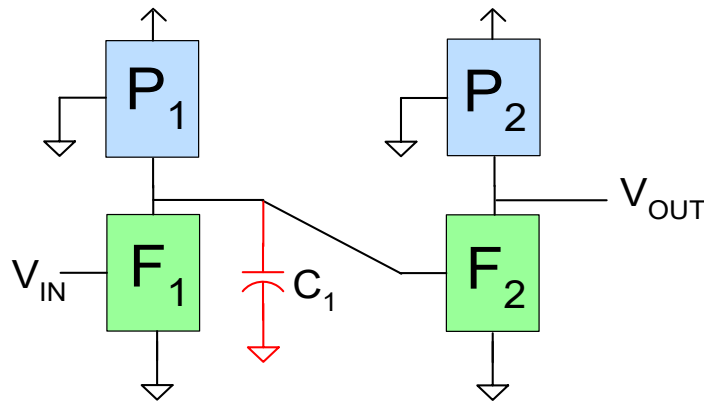


Will not provide  
compensation!

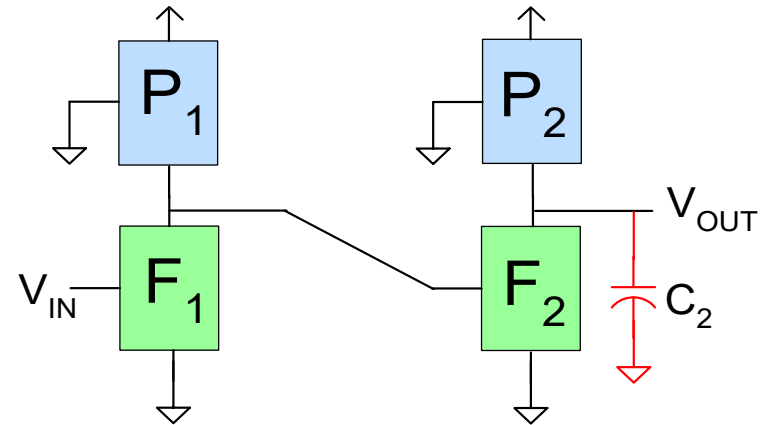


# Compensation of Basic Two-Stage Cascade

(shown for single input, single output but applicable to differential as well)



Internal Node Compensated



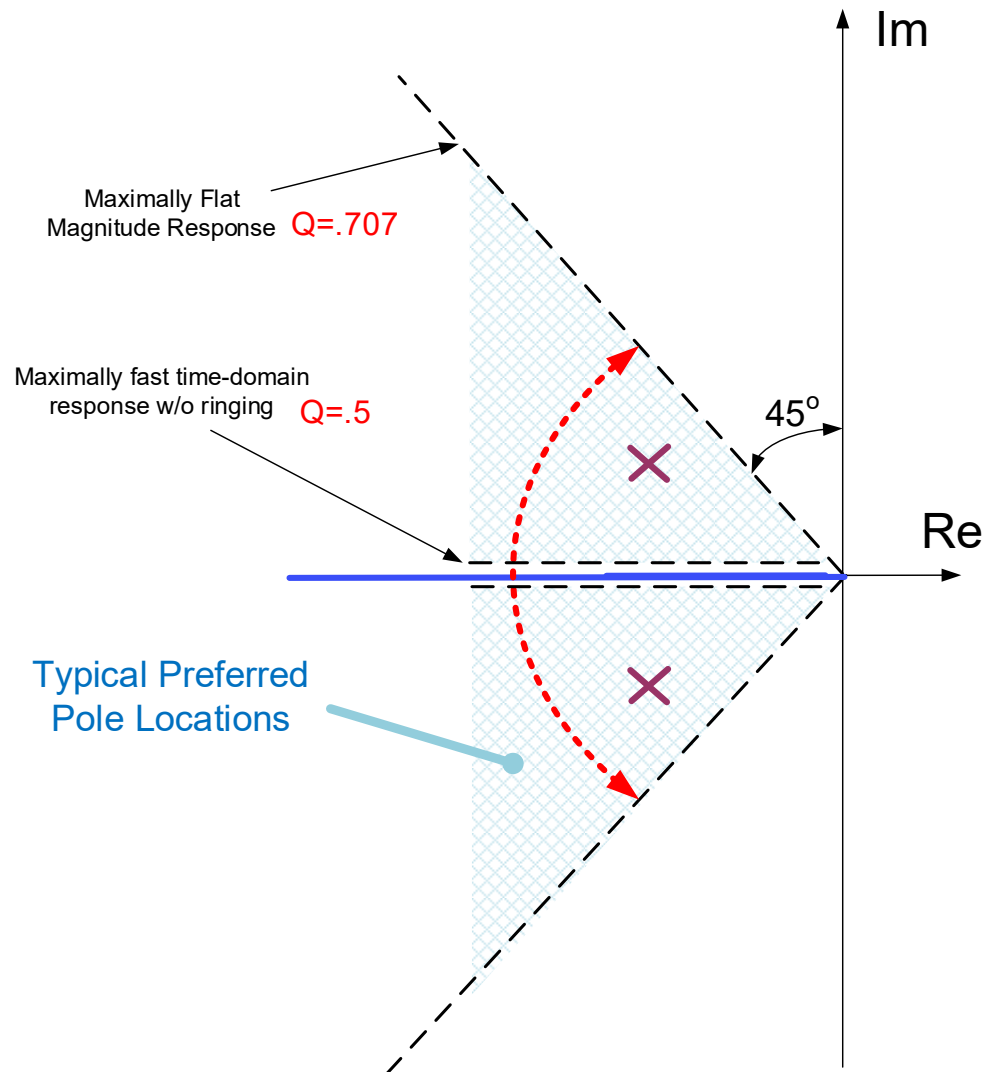
Output Compensated

- Modest variants of the compensation principle are often used
- Internal Node compensated creates the dominant pole on the internal node
- Output compensated creates the dominant pole on the external node
- Output compensated often termed “self-compensated”
- Internally Compensated denotes putting compensation circuit on-chip

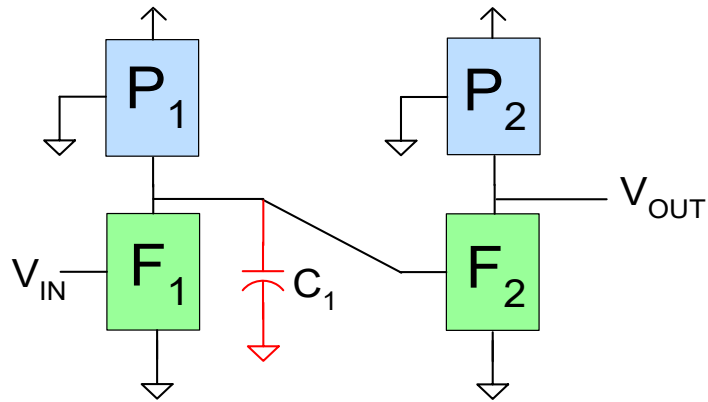
Everything else is just details !!

# Common Compensation Goal

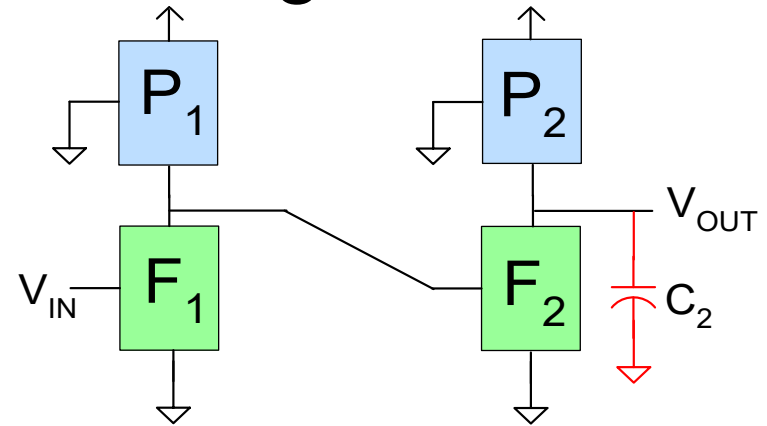
Typical Target Closed-loop Pole Locations for Feedback Amplifiers



# Compensation of Basic Two-Stage Cascade

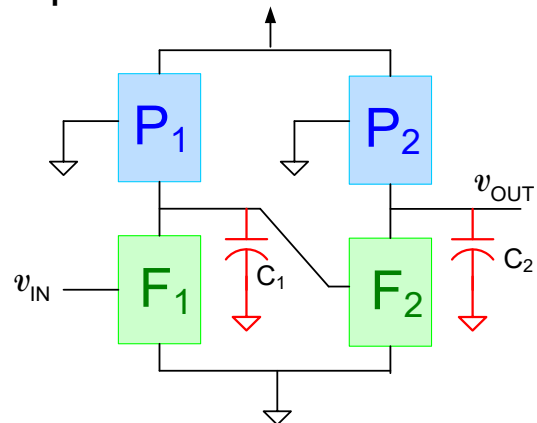


Internally Compensated



Output Compensated

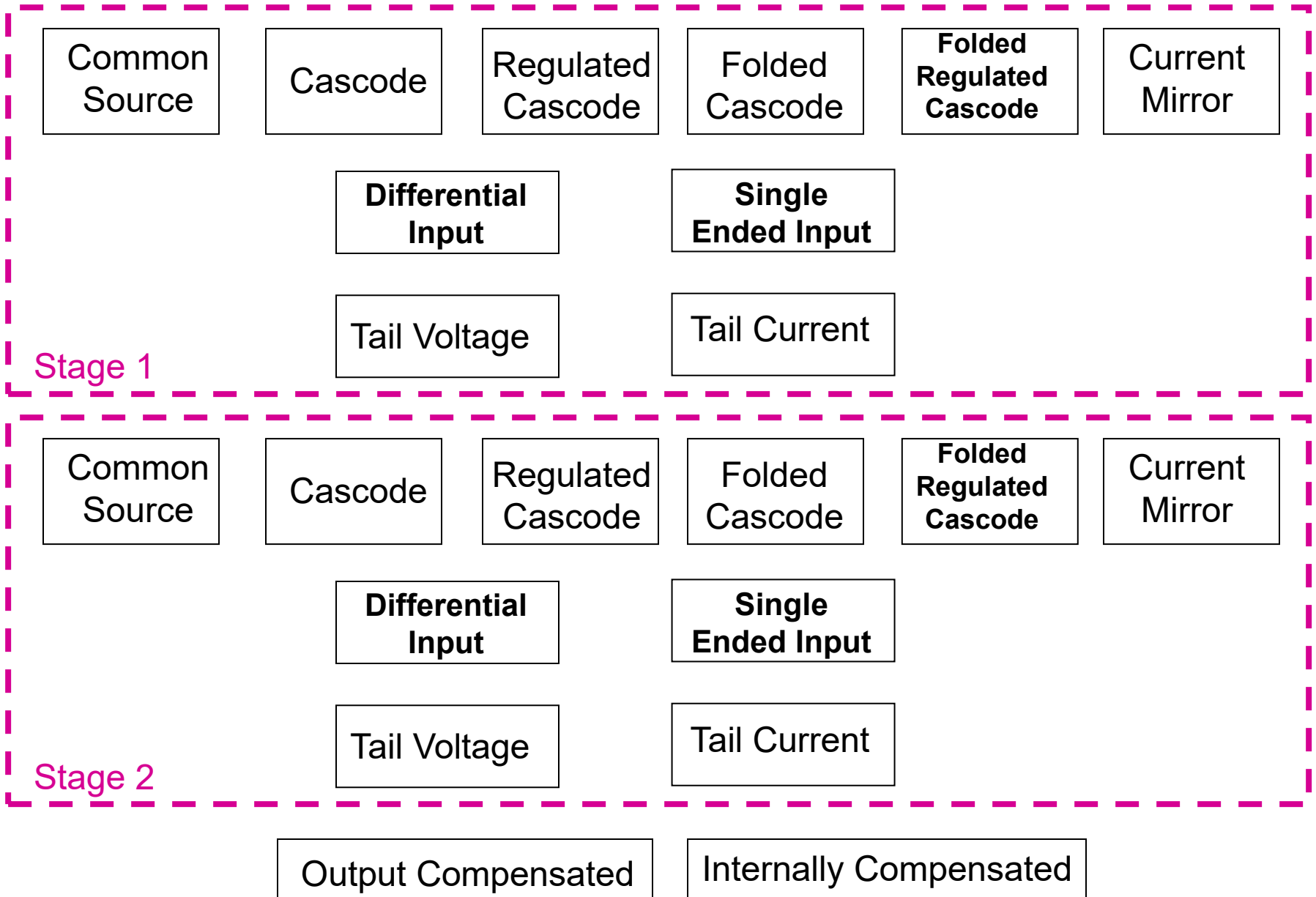
Question: Would double compensation be even better?



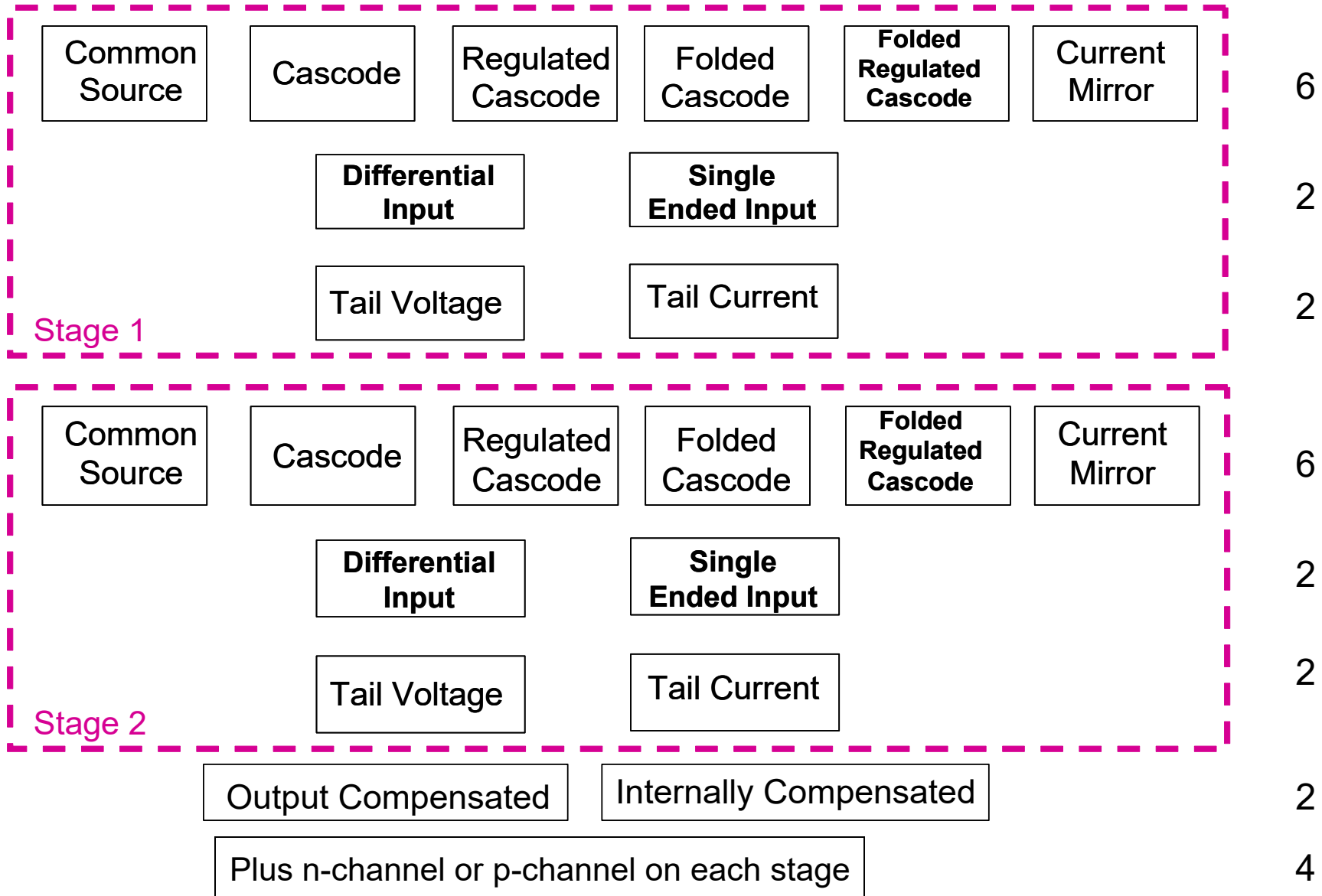
Double Compensated

No! A second compensation capacitor would move the open-loop poles back together !

# Two-stage Architectural Choices

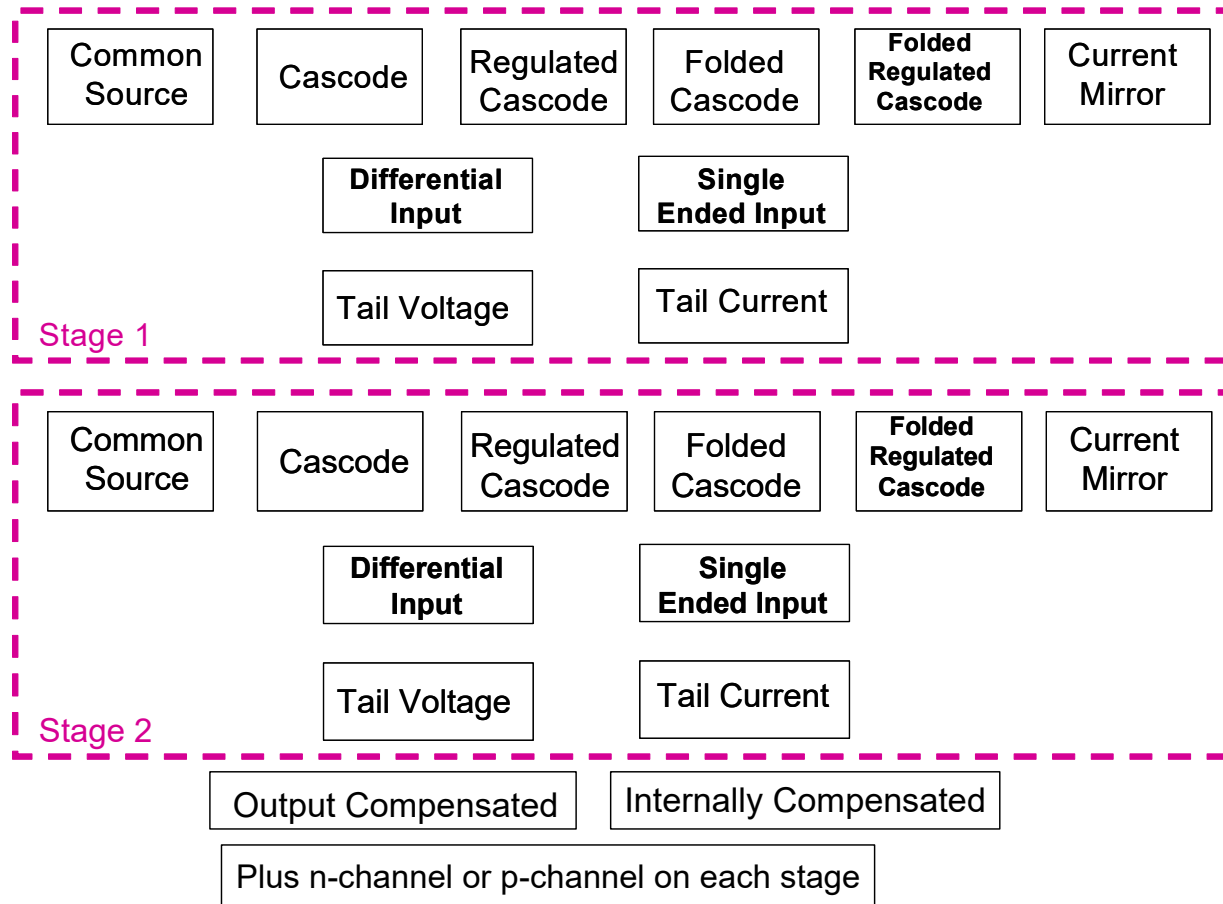


# Two-stage Architectural Choices



2304 Choices !!!

# Two-stage Architectural Choices



Which of these 2304 choices can be used to build a good op amp?

**All of them !!**



Stay Safe and Stay Healthy !

**End of Lecture 13**